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二阶比估计

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摘　要：给出了二阶比估计量的偏差、估计量的均方误差以及均方误差的估计.

关键词：二阶抽样；比估计；均方误差；均方误差估计量

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Ratio Estimate in Two-Stage Sampling

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Abstract: For the ratio estimate in two-stage sampling, the bias and mean square errors(MES) of the estimater value have been obtained, and the estimater value of the MSE has been given.

Key words: two-stage sampling; the ratio estimate; MSE; the estimat value of the MSE

一阶单元大小差别较大时的二阶抽样，采用简单随机抽样估计的效果一般一不好.为了提高一阶单元大小不等时二阶抽样的估计精度，常把一阶单元大小作为辅助变量而采用二阶比估计[1,2].本文则进一步给出了一般辅助变量时二阶比估计量的偏差，估计量的均方误差以及均方误差的估计.而通常以一阶单元大小作为辅助变量的二阶比估计，则是二阶比估计的特例.

设总体的 N 个二阶单元划分为 K 个一阶单元，第 i 个一阶单元包含 N_i 个二阶单元.第 i 个一阶单元中第 j 个二阶单元的标志值用 Y_{ij} 表示，$i=1,2,\cdots,K$；$j=1,2,\cdots,N_i$，$N=\sum\limits_{i=1}^{K}N_i$.同时，第 i 个一阶单元的辅助变量用 X_i 表示，$i=1,2,\cdots,K$.记

$$\bar{X}=\frac{1}{K}\sum_{i=1}^{K}X_i,\quad \bar{Y}_i=\frac{1}{N_i}\sum_{j=1}^{N_i}Y_{ij}=\frac{Y_i}{N_i}$$

$$W_i=\frac{N_i}{N},\quad S_i^2=\frac{1}{N_i-1}\sum_{j=1}^{N_i}(Y_{ij}-\bar{Y}_i)^2$$

$$\bar{\bar{Y}}=\frac{1}{N}\sum_{i=1}^{K}\sum_{j=1}^{N_i}Y_{ij}=\sum_{i=1}^{K}\frac{N_i}{N}\bar{Y}_i=\sum_{i=1}^{K}W_i\bar{Y}_i,$$
$$R=\frac{\bar{\bar{Y}}}{\bar{\bar{X}}}\tag{1}$$

设采用简单随机抽样方法从 K 个一阶单元中抽取 k 个，再从第 i 个被抽中的一阶单元中抽取 n_i 个二阶单元，共有 $n=\sum\limits_{i=1}^{k}n_i$ 个二阶单元组成样本.第 i 个取到的一阶单元中第 j 个取到的二阶单元标志值用 y_{ij} 表示，而相应的辅助变量用 x_i 表示，$i=1,2,\cdots,k$；$j=1,2,\cdots,n_i$.记

$$\bar{x}=\frac{1}{k}\sum_{i=1}^{k}x_i,\qquad \bar{y}_i\ \frac{1}{n_i}\sum_{j=1}^{n_i}y_{ij},$$
$$\bar{\bar{y}}=\frac{K}{k}\sum_{i=1}^{k}W_i\bar{y}_i,\qquad \bar{\bar{y}}_{TR}=\frac{\bar{\bar{y}}}{\bar{x}}\bar{X}=\hat{R}\bar{X},\tag{2}$$
$$\hat{R}=\frac{\bar{\bar{y}}}{\bar{x}},\qquad s_i^2=\frac{1}{n_i-1}\sum_{j=1}^{n_i}(y_{ij}-\bar{y}_i)^2$$

二阶比估计是用 $\bar{\bar{y}}_{TR}$ 作为 $\bar{\bar{Y}}$ 的估计.

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1 估计量的偏差

定理 1 设 $X_i>0,\ i=1,2,\cdots,K;\ \bar{\bar{Y}}>0$. 则对两个阶段的抽样都是简单随机的二阶抽样，有.

$$E(\bar{\bar{y}}_{TR})=\bar{\bar{Y}}+O(\frac{1}{k}) \tag{3}$$

证 根据(2)式，则

$$\bar{\bar{y}}_{TR}=\bar{\bar{Y}}+(\bar{\bar{y}}-R\,\bar{x})-\frac{(\bar{\bar{y}}-R\,\bar{x})(\bar{x}-\bar{X})}{\bar{x}}$$

由于[3]

$$E(\bar{\bar{y}})=\bar{\bar{Y}},\quad E(\bar{x})=\bar{X},$$

$$E(\bar{\bar{y}}-\bar{\bar{Y}})^2=O(\frac{1}{k}),\quad E(\bar{x}-\bar{X})^2=O(\frac{1}{k})$$

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$$E(\bar{\bar{y}}_{TR})=\bar{\bar{Y}}-E[\frac{(\bar{\bar{y}}-R\,\bar{x})(\bar{x}-\bar{X})}{\bar{x}}]$$

记 $X_0=\min\{X_1,X_2,\cdots,X_K\}$，于是 $\bar{x}\geqslant X_0>0$，从而

$$|E(\bar{\bar{y}}_{TR})-\bar{\bar{Y}}|\leqslant E|\frac{(\bar{\bar{y}}-R\,\bar{x})(\bar{x}-\bar{X})}{\bar{x}}|\leqslant$$
$$\frac{1}{X_0}E|(\bar{\bar{y}}-\bar{\bar{Y}})(\bar{x}-\bar{X})-R(\bar{x}-\bar{X})^2|\leqslant$$
$$\frac{1}{X_0}\{E|(\bar{\bar{y}}-\bar{\bar{Y}})(\bar{x}-\bar{X})|+RE(\bar{x}-\bar{X})^2\}\leqslant$$
$$\frac{1}{X_0}\{\sqrt{E(\bar{\bar{y}}-\bar{\bar{Y}})^2}\sqrt{E(\bar{x}-\bar{X})^2}+$$
$$RE(\bar{x}-\bar{X})^2\}=O(\frac{1}{k})$$

特别，若取 $X_i=N_i,\ i=1,2,\cdots,K$. 根据(2)式即得把 N_i 作为辅助变量时的二阶估计量为

$$\bar{\bar{y}}_R=\frac{\sum_{i=1}^{k}N_i\bar{y}_i}{\sum_{i=1}^{k}N_i} \tag{4}$$

推论 1 在定理 1 的条件下，有

$$E(\bar{\bar{y}}_R)=\bar{\bar{Y}}+O(\frac{1}{k}) \tag{5}$$

2 估计量的均方误差

定理 2 在定理 1 的条件下，$\bar{\bar{y}}_{TR}$的均方误差为

$$\mathrm{MSE}(\bar{\bar{y}}_{TR})=\frac{K}{k}\sum_{\alpha=1}^{K}W_\alpha{}^2(\frac{1}{n_\alpha}-\frac{1}{N_\alpha})S_\alpha^2+$$
$$(\frac{1}{k}-\frac{1}{K})\frac{K^2}{K-1}\sum_{\alpha=1}^{K}(W_\alpha\bar{Y}_\alpha-\frac{R}{K}X_\alpha)^2+$$
$$O(\frac{1}{k^2}) \tag{6}$$

证 因为[1]

$$V(\bar{\bar{y}}_{TR})=E_1[V_2(\bar{\bar{y}}_{TR})]+V_1[E_2(\bar{\bar{y}}_{TR})]$$

其中 E_1 和 V_1 分别表示对第一阶抽样求平均和方差，而 E_2 和 V_2 分别表示在给定的一阶单元内对第二阶抽样求平均和方差. 其中

$$V_2(\bar{\bar{y}}_{TR})=\frac{K^2\bar{X}^2}{k^2\,\bar{x}^2}V_2(\sum_{i=1}^{k}W_i\bar{y}_i)=$$
$$\frac{K^2\bar{X}^2}{k^2\,\bar{x}^2}\sum_{i=1}^{k}W_i{}^2(\frac{1}{n_i}-\frac{1}{N_i})S_i{}^2$$

由于

$$\frac{\bar{X}^2}{\bar{x}^2}=1-\frac{2(\bar{x}-\bar{X})}{\bar{X}}+\frac{2(\bar{x}-\bar{X})^2}{\bar{x}\bar{X}}+\frac{(\bar{x}-\bar{X})^2}{\bar{x}^2}$$

记

$$u_i=(\frac{N_i}{M_0})^2(\frac{1}{n_i}-\frac{1}{N_i})S_i{}^2,\quad M_0=\frac{N}{K},$$

$$\bar{u}=\frac{1}{k}\sum_{i=1}^{k}u_i=\frac{K^2}{k}\sum_{i=1}^{k}W_i{}^2(\frac{1}{n_i}-\frac{1}{N_i})S_i{}^2$$

则

$$V_2(\bar{\bar{y}}_{TR})=[1-\frac{2(\bar{x}-\bar{X})}{\bar{X}}+\frac{2(\bar{x}-\bar{X})^2}{\bar{x}\bar{X}}+$$
$$\frac{(\bar{x}-\bar{X})^2}{\bar{x}^2}]\frac{\bar{u}}{k}$$

而

$$E_1(\bar{u})=\bar{U}=K\sum_{\alpha=1}^{K}W_\alpha{}^2(\frac{1}{n_\alpha}-\frac{1}{N_\alpha})S_\alpha^2$$

$$E_1[\bar{u}(\bar{x}-\bar{X})]=E_1(\bar{u}-\bar{U})(\bar{x}-\bar{X})=O(\frac{1}{k})$$

$$E_1[\bar{u}(\bar{x}-\bar{X})^2]=O(\frac{1}{k})$$

于是

$$E_1(\frac{\bar{u}}{k})=\frac{K}{k}\sum_{\alpha=1}^{K}W_\alpha{}^2(\frac{1}{n_\alpha}-\frac{1}{N_\alpha})S_\alpha^2$$

$$E_1[\frac{2\bar{u}(\bar{x}-\bar{X})}{k\bar{X}}]=O(\frac{1}{k^2})$$

$$E_1[\frac{2\bar{u}(\bar{x}-\bar{X})^2}{k\,\bar{x}\bar{X}}]=O(\frac{1}{k^2})$$

$$E_1[\frac{\bar{u}(\bar{x}-\bar{X})^2}{k\,\bar{x}^2}]=O(\frac{1}{k^2})$$

所以

$$E_1[V_2(\bar{\bar{y}}_{TR})]=\frac{K}{k}\sum_{\alpha=1}^{K}W_\alpha{}^2(\frac{1}{n_\alpha}-\frac{1}{N_\alpha})S_\alpha^2+$$
$$O(\frac{1}{k^2})$$

记

$$\bar{y}=\frac{1}{k}\sum_{i=1}^{k}Y_i$$

则

$$E_2(\bar{\bar{y}}_{TR})=E_2(\frac{\bar{\bar{y}}}{\bar{x}}\bar{X})=\frac{\bar{X}}{\bar{x}}E_2(\frac{K}{k}\sum_{i=1}^{k}W_i\bar{y}_i)=$$
$$\frac{K}{k}\frac{\bar{X}}{\bar{x}}\sum_{i=1}^{k}W_i\bar{Y}_i=\frac{K}{N}\frac{\bar{X}}{\bar{x}}(\frac{1}{k}\sum_{i=1}^{k}Y_i)=\frac{K}{N}\frac{\bar{X}}{\bar{x}}\bar{y}$$

于是[2]

$$V_1[E_2(\bar{\bar{y}}_{TR})]=\frac{K^2\bar{X}^2}{N^2}V_1(\frac{\bar{y}}{\bar{x}})=$$

$$\frac{K^2\overline{X}^2}{N^2}[\frac{1}{\bar{\bar{X}}^2}(\frac{1}{k}-\frac{1}{K})\frac{1}{K-1}\sum_{\alpha=1}^{K}(Y_\alpha-R_0Y_\alpha)^2+$$

$$O(\frac{1}{k^2})]=(\frac{1}{k}-\frac{1}{K})\frac{K^2}{N^2(K-1)}$$

$$\sum_{\alpha=1}^{K}(Y_\alpha-R_0Y_\alpha)^2+O(\frac{1}{k^2})$$

其中

$$R_0=\frac{\frac{1}{K}\sum_{\alpha=1}^{K}Y_\alpha}{\frac{1}{K}\sum_{\alpha=1}^{K}X_\alpha}=\frac{\frac{N}{K}\sum_{\alpha=1}^{K}W_\alpha\overline{Y}_\alpha}{\bar{\bar{X}}}=\frac{N}{K}R$$

即

$$V_1[E_2(\bar{\bar{y}}_{TR})]=(\frac{1}{k}-\frac{1}{K})\frac{K^2}{N^2(K-1)}$$

$$\sum_{\alpha=1}^{K}(Y_\alpha-\frac{N}{K}RX_\alpha)^2+O(\frac{1}{k^2})=$$

$$(\frac{1}{k}-\frac{1}{K})\frac{K^2}{K-1}\sum_{\alpha=1}^{K}(W_\alpha\overline{Y}_\alpha-\frac{R}{K}X_\alpha)^2+O(\frac{1}{k^2})$$

又由于

$$\mathrm{MSE}(\bar{\bar{y}}_{TR})=V(\bar{\bar{y}}_{TR})+O(\frac{1}{k^2})$$

从而

$$\mathrm{MSE}(\bar{\bar{y}}_{TR})=\frac{K}{k}\sum_{\alpha=1}^{K}W_\alpha{}^2(\frac{1}{n_\alpha}-\frac{1}{N_\alpha})S_\alpha^2+$$

$$(\frac{1}{k}-\frac{1}{K})\frac{K^2}{K-1}\sum_{\alpha=1}^{K}(W_\alpha\overline{Y}_\alpha-\frac{R}{K}X_\alpha)^2+O(\frac{1}{k^2})$$

根据(6)式，即得以 N_i 为辅助变量时二阶比估计量的均方误差.

推论 2　在定理 1 的条件下，$\bar{\bar{y}}_R$ 的均方误差为

$$\mathrm{MSE}(\bar{\bar{y}}_{TR})=\frac{K}{k}\sum_{\alpha=1}^{K}W_\alpha{}^2(\frac{1}{n_\alpha}-\frac{1}{N_\alpha})S_\alpha^2+$$

$$(\frac{1}{k}-\frac{1}{K})\frac{K^2}{K-1}\sum_{\alpha=1}^{K}W_\alpha{}^2(\overline{Y}_\alpha-\bar{\bar{Y}})^2+O(\frac{1}{k^2})\qquad(7)$$

3　均方误差的估计

定理 3　在定理 1 的条件下，有

$$E[\mathrm{mse}(\bar{\bar{y}}_{TR})]=\frac{K}{k}\sum_{\alpha=1}^{K}W_\alpha{}^2(\frac{1}{n_\alpha}-\frac{1}{N_\alpha})S_\alpha^2+$$

$$(\frac{1}{k}-\frac{1}{K})\frac{K^2}{K-1}\sum_{\alpha=1}^{K}(W_\alpha\overline{Y}_\alpha-\frac{R}{K}X_\alpha)^2+O(\frac{1}{k^2})\qquad(8)$$

其中

$$\mathrm{mse}(\bar{\bar{y}}_{TR})=\frac{K}{k}\sum_{i=1}^{k}W_i{}^2(\frac{1}{n_i}-\frac{1}{N_i})s_i^2+$$

$$(\frac{1}{k}-\frac{1}{K})\frac{K^2}{k-1}\sum_{i=1}^{k}(W_i\bar{y}_i-\frac{\hat{R}}{K}x_i)^2\qquad(9)$$

证　由于

$$E[\frac{K}{k}\sum_{i=1}^{k}W_i{}^2(\frac{1}{n_i}-\frac{1}{N_i})s_i^2]=$$

$$\frac{k}{K}E_1\{E_2[\sum_{i=1}^{k}W_i{}^2(\frac{1}{n_i}-\frac{1}{N_i})s_i^2]\}=$$

$$\frac{K}{k}E_1[\sum_{i=1}^{k}W_i{}^2(\frac{1}{n_i}-\frac{1}{N_i})S_i^2]=$$

$$\sum_{\alpha=1}^{K}W_\alpha^2(\frac{1}{n_\alpha}-\frac{1}{N_\alpha})S_\alpha^2\qquad(10)$$

而

$$K^2\sum_{i=1}^{k}(W_i\bar{y}_i-\frac{\hat{R}}{K}x_i)^2=K^2\sum_{i=1}^{k}W_i{}^2\bar{y}_i{}^2-$$

$$2K\hat{R}\sum_{i=1}^{k}W_i\bar{y}_ix_i+\hat{R}^2\sum_{i=1}^{k}x_i{}^2\qquad(11)$$

其中

$$E(K^2\sum_{i=1}^{k}W_i{}^2\bar{y}_i{}^2)=K^2E_1[E_2(\sum_{i=1}^{k}W_i{}^2\bar{y}_i{}^2)]=$$

$$K^2E_1\{\sum_{i=1}^{k}W_i{}^2[V_2(\bar{y}_i)+E_2{}^2(\bar{y}_i)]\}=$$

$$K^2E_1[\sum_{i=1}^{k}W_i{}^2(\frac{1}{n_i}-\frac{1}{N_i})S_i^2+\sum_{i=1}^{k}W_i{}^2\overline{Y}_i^2]=$$

$$kK\sum_{\alpha=1}^{K}W_\alpha{}^2(\frac{1}{n_\alpha}-\frac{1}{N_\alpha})S_\alpha^2+kK\sum_{\alpha=1}^{K}W_\alpha{}^2\overline{Y}_\alpha^2\qquad(12)$$

由于

$$E_2(2K\hat{R}\sum_{i=1}^{k}W_i\bar{y}_ix_i)=$$

$$\frac{2K^2}{k\bar{x}}E_2(\sum_{i=1}^{k}W_i\bar{y}_i\sum_{i=1}^{k}W_i\bar{y}_ix_i)=$$

$$\frac{2K^2}{k\bar{x}}E_2(\sum_{i=1}^{k}W_i^2\bar{y}_i{}^2x_i+\sum_{i\neq j}^{k}W_i\bar{y}_iW_j\bar{y}_jx_j)=$$

$$\frac{2K^2}{k\bar{x}}\{\sum_{i=1}^{k}W_i{}^2x_i[V_2(\bar{y}_i)+E_2{}^2(\bar{y}_i)]+$$

$$\sum_{i\neq j}^{k}W_iW_jx_jE_2(\bar{y}_i)E_2(\bar{y}_j)\}=$$

$$\frac{2K^2}{k\bar{x}}[\sum_{i=1}^{k}W_i{}^2x_i(\frac{1}{n_i}-\frac{1}{N_i})S_i^2+$$

$$\sum_{i=1}^{k}W_i\overline{Y}_i\sum_{i=1}^{k}W_i\overline{Y}_ix_i]=$$

$$\frac{2K^2}{k}(\frac{1}{\bar{\bar{X}}}-\frac{\bar{x}-\bar{\bar{X}}}{\bar{x}\bar{\bar{X}}})[\sum_{i=1}^{k}W_i{}^2x_i(\frac{1}{n_i}-\frac{1}{N_i})S_i^2+$$

$$\sum_{i=1}^{k}W_i\overline{Y}_i\sum_{i=1}^{k}W_i\overline{Y}_ix_i]$$

于是

$$E(2K\hat{R}\sum_{i=1}^{k}W_i\bar{y}_ix_i)=$$

$$\frac{2K^2}{k\bar{\bar{X}}}E_1(\sum_{i=1}^{k}W_i\overline{Y}_i\sum_{i=1}^{k}W_i\overline{Y}_ix_i)+O(1)=$$

$$\frac{2K^2}{k\bar{\bar{X}}}E_1(\sum_{i=1}^{k}W_i^2\overline{Y}_i{}^2x_i+\sum_{i\neq j}^{k}W_i\overline{Y}_iW_j\overline{Y}_jx_j)+O(1)=$$

$$\frac{2K^2}{k\bar{\bar{X}}}[\frac{k(K-k)}{K(K-1)}\sum_{\alpha=1}^{K}W_\alpha{}^2\overline{Y}_\alpha{}^2X_\alpha+$$

$$\frac{k(k-1)}{K(K-1)}\bar{\bar{Y}}\sum_{\alpha=1}^{K}W_\alpha\overline{Y}_\alpha X_\alpha]+O(1)=$$

$$\frac{2K(k-1)}{K-1}R\sum_{\alpha=1}^{K}W_\alpha\overline{Y}_\alpha X_\alpha+O(1)\qquad(13)$$

且

$$E_2(\hat{R}^2\sum_{i=1}^{k}x_i{}^2)=\frac{K^2\sum_{i=1}^{k}x_i{}^2}{k^2\bar{x}^2}E_2(\sum_{i=1}^{k}W_i\bar{y}_i)^2=$$

$$\frac{K^2\sum_{i=1}^{k}x_i{}^2}{k^2\bar{x}^2}[\sum_{i=1}^{k}W_i^2(\frac{1}{n_i}-\frac{1}{N_i})S_i^2+(\sum_{i=1}^{k}W_i\overline{Y}_i)^2]$$

又由于

$$\frac{1}{\bar{x}^2}=\frac{1}{\overline{X}^2}-(\frac{1}{\overline{X}}+\frac{1}{\bar{x}})\frac{\bar{x}-\overline{X}}{\bar{x}\overline{X}}$$

则

$$E(\hat{R}^2\sum_{i=1}^{k}x_i{}^2)=$$

$$E_1\{\frac{K^2\sum_{i=1}^{k}x_i{}^2}{k^2\overline{X}^2}[\sum_{i=1}^{k}W_i{}^2(\frac{1}{n_i}-\frac{1}{N_i})S_i^2+$$

$$(\sum_{i=1}^{k}W_i\overline{Y}_i)^2]\}-E_1\{\frac{K^2\sum_{i=1}^{k}x_i{}^2}{k^2}(\frac{1}{\overline{X}}+\frac{1}{\bar{x}})\frac{\bar{x}-\overline{X}}{\bar{x}\overline{X}}$$

$$[\sum_{i=1}^{k}W_i{}^2(\frac{1}{n_i}-\frac{1}{N_i})S_i^2+(\sum_{i=1}^{k}W_i\overline{Y}_i)^2]\}$$

其中

$$E_1[\frac{K^2}{k^2}\sum_{i=1}^{k}x_i{}^2\sum_{i=1}^{k}W_i{}^2(\frac{1}{n_i}-\frac{1}{N_i})S_i^2]=O(1)$$

$$E_1[\frac{K^2}{k^2\overline{X}^2}\sum_{i=1}^{k}x_i{}^2(\sum_{i=1}^{k}W_i\overline{Y}_i)^2]=$$

$$[\frac{K^2}{k^2\overline{X}^2}E_1[\sum_{i=1}^{k}W_i{}^2\overline{Y}_i^2x_i{}^2+\sum_{i\neq j}^{k}W_i{}^2\overline{Y}_i^2x_j{}^2+$$

$$2\sum_{i\neq j}^{k}W_i\overline{Y}_ix_i{}^2W_j\overline{Y}_j+\sum_{i\neq j\neq l}^{k}W_i\overline{Y}_iW_j\overline{Y}_jx_l{}^2]=$$

$$\frac{K^2}{k^2\overline{X}^2}[\frac{k}{K}\sum_{\alpha=1}^{K}W_\alpha{}^2\overline{Y}_\alpha^2X_\alpha{}^2+\frac{k(k-1)}{K(K-1)}\sum_{\alpha\neq\beta}^{K}W_\alpha{}^2\overline{Y}_\alpha^2X_\beta{}^2+$$

$$\frac{2k(k-1)}{K(K-1)}\sum_{\alpha\neq\beta}^{K}W_\alpha\overline{Y}_\alpha X_\alpha{}^2W_\beta\overline{Y}_\beta+\frac{k(k-1)(k-2)}{K(K-1)(K-2)}$$

$$\sum_{\alpha\neq\beta\neq\gamma}^{K}W_\alpha\overline{Y}_\alpha W_\beta\overline{Y}_\beta X_\gamma{}^2]=\frac{K^2}{k^2\overline{X}^2}\{\frac{k}{K}\sum_{\alpha=1}^{K}W_\alpha{}^2\overline{Y}_\alpha^2X_\alpha{}^2+$$

$$\frac{k(k-1)}{K(K-1)}[\sum_{\alpha=1}^{K}W_\alpha{}^2\overline{Y}_\alpha^2\sum_{\alpha=1}^{K}X_\alpha{}^2-\sum_{\alpha=1}^{K}W_\alpha{}^2\overline{Y}_\alpha^2X_\alpha{}^2]+$$

$$\frac{2k(k-1)}{K(K-1)}[\sum_{\alpha=1}^{K}W_\alpha\overline{Y}_\alpha X_\alpha{}^2\sum_{\alpha=1}^{K}W_\alpha\overline{Y}_\alpha-$$

$$\sum_{\alpha=1}^{K}W_\alpha{}^2\overline{Y}_\alpha{}^2X_\alpha{}^2]+\frac{k(k-1)(k-2)}{K(K-1)(K-2)}$$

$$\{[(\sum_{\alpha=1}^{K}W_\alpha\overline{Y}_\alpha)^2-\sum_{\alpha=1}^{K}W_\alpha{}^2\overline{Y}_\alpha{}^2]\sum_{\alpha=1}^{K}X_\alpha{}^2-$$

$$2[\sum_{\alpha=1}^{K}W_\alpha\overline{Y}_\alpha X_\alpha{}^2\sum_{\alpha=1}^{K}W_\alpha\overline{Y}_\alpha-\sum_{\alpha=1}^{K}W_\alpha{}^2\overline{Y}_\alpha{}^2X_\alpha{}^2]\}\}=$$

$$\frac{K(k-1)(k-2)}{k(K-1)(K-2)}R^2\sum_{\alpha=1}^{K}X_\alpha{}^2+O(1)$$

$$E_1\{\frac{K^2\sum_{i=1}^{k}x_i{}^2}{k^2}(\frac{1}{\overline{X}}+\frac{1}{\bar{x}})\frac{\bar{x}-\overline{X}}{\bar{x}\overline{X}}$$

$$[\sum_{i=1}^{k}W_i{}^2(\frac{1}{n_i}-\frac{1}{N_i})S_i^2+(\sum_{i=1}^{k}W_i\overline{Y}_i)^2]\}=O(1)$$

从而

$$E(\hat{R}^2\sum_{i=1}^{k}x_i{}^2)=$$

$$\frac{K(k-1)(k-2)}{k(K-1)(K-2)}R^2\sum_{\alpha=1}^{K}X_\alpha{}^2+O(1) \tag{14}$$

根据(11)、(12)、(13)和(14)式，则

$$E[(\frac{1}{k}-\frac{1}{K})\frac{K^2}{k-1}\sum_{i=1}^{k}(W_i\bar{y}_i-\frac{\hat{R}}{K}x_i)^2]=$$

$$\frac{K-k}{k}\sum_{\alpha=1}^{K}W_\alpha{}^2(\frac{1}{n_\alpha}-\frac{1}{N_\alpha})S_\alpha^2+$$

$$(\frac{1}{k}-\frac{1}{K})\frac{K^2}{K-1}\sum_{\alpha=1}^{K}(W_\alpha\overline{Y}_\alpha-\frac{R}{K}X_\alpha)^2+O(\frac{1}{k^2}) \tag{15}$$

再按(9)、(10)和(15)式，即得

$$E[\mathrm{mse}(\bar{\bar{y}}_{TR})]=\frac{K}{k}\sum_{\alpha=1}^{K}W_\alpha{}^2(\frac{1}{n_\alpha}-\frac{1}{N_\alpha})S_\alpha^2+$$

$$(\frac{1}{k}-\frac{1}{K})\frac{K^2}{K-1}\sum_{\alpha=1}^{K}(W_\alpha\overline{Y}_\alpha-\frac{R}{K}X_\alpha)^2+O(\frac{1}{k^2})$$

推论 3　在定理 1 的条件下，有

$$E[\mathrm{mse}(\bar{\bar{y}}_R)]=\frac{K}{k}\sum_{\alpha=1}^{K}W_\alpha{}^2(\frac{1}{n_\alpha}-\frac{1}{N_\alpha})S_\alpha^2+$$

$$(\frac{1}{k}-\frac{1}{K})\frac{K^2}{K-1}\sum_{\alpha=1}^{K}W_\alpha{}^2(\overline{Y}_\alpha-\overline{\overline{Y}})^2+O(\frac{1}{k^2}) \tag{16}$$

其中

$$\mathrm{mse}(\bar{\bar{y}}_R)=\frac{K}{k}\sum_{i=1}^{k}W_i{}^2(\frac{1}{n_i}-\frac{1}{N_i})s_i^2+$$

$$(\frac{1}{k}-\frac{1}{K})\frac{K^2}{k-1}\sum_{i=1}^{k}W_i(\bar{y}_i-\bar{\bar{y}}_R)^2 \tag{17}$$

4　结　语

为了提高一阶单元大小不等时的二阶抽样估计精度，目前常采用把一阶单元大小作为辅助变量的二阶比估计．作者进一步研究了采用一般辅助变量时的二阶比估计，给出了这种二阶比估计量的偏差和均方误差以及均方误差的估计量．而以一阶单元大小作为辅助变量的二阶比估计，则是作者研究的一般辅助变量时二阶比估计的特例．

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